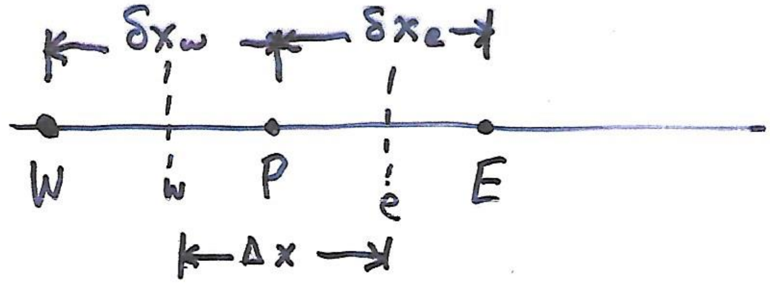
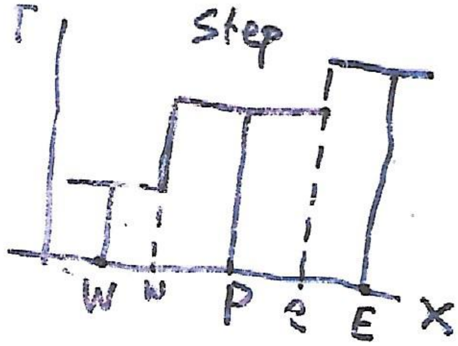
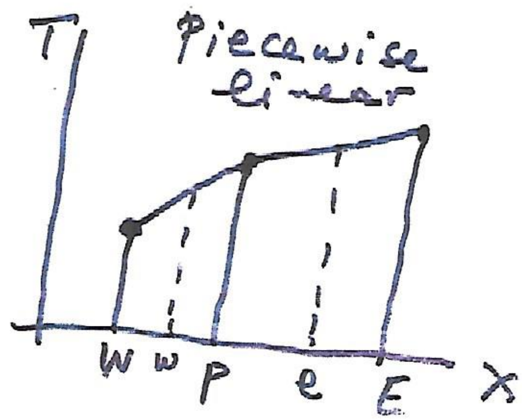


$$\frac{d}{dx}\left(\kappa \frac{dT}{\partial x}\right) + S = 0$$

Integrate:

$$\int_w^e \frac{d}{dx}\left(\kappa \frac{dT}{dx}\right) + s = \left(\kappa \frac{dT}{dx} A\right)_e - \left(\kappa \frac{dT}{dx} A\right)_w + \int_w^e s\, dx\, A = 0$$





$$K_e A (T_E - T_P)/(\delta x)_e - K_w A (T_P - T_W)/(\delta x)_w + \bar{S} \Delta x A$$

$$A \quad \frac{K_w A}{(\delta x)_w}\Big] T_P = \frac{K_e A}{(\delta x)_e} T_E + \frac{K_w A}{(\delta x)_w} T_W + \bar{S} \Delta x A$$

$$a_P T_P = a_E T_E + a_W T_W + b$$

$$a_E = \frac{K_e A_e}{(\delta x)_e} \qquad a_W = \frac{K_w A_w}{(\delta x)_w}$$

$$\bar{S} \Delta V = S_u + S_P \phi_P$$

$$\underline{\mathrm{div}(\Gamma \,\mathrm{grad}\,\phi) + S_\phi = 0}$$

$$\int_{CV} \mathrm{div}(\Gamma \,\mathrm{grad}\,\phi)\, dV + \int_{CV} S_\phi \, dV = \int_A \underline{n} \,(\Gamma \,\mathrm{grad}\,\phi)\, dA + \int_{CV} S_\phi \, dV$$

1-dimensional

$$\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) + S = 0$$

area

$$\int_{\Delta V} \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) dV + \int_{\Delta V} S\, dV = \left(\Gamma A \frac{d\phi}{dx}\right)_e - \left(\Gamma A \frac{d\phi}{dx}\right)_w + \bar{S}\,\Delta V$$

$$\Gamma_w = (\Gamma_W + \Gamma_P)/2 \quad , \quad \Gamma_e = (\Gamma_P + \Gamma_E)/2$$

$$\left(\Gamma A \frac{d\phi}{dx}\right)_e = \Gamma_e A_e \frac{\phi_E - \phi_P}{\delta x_{PE}} \quad , \quad \left(\Gamma A \frac{d\phi}{dx}\right)_w = \Gamma_w A_w \left(\frac{\phi_P - \phi_W}{\delta x_{WP}}\right)$$

$$\bar{S}\,\Delta V = S_U + S_P \phi_P$$

collect terms, ϕ_P on the left, rest on the right

$$\left(\frac{\Gamma_e}{\delta x_{PE}} A_e + \frac{\Gamma_w}{\delta x_{WP}} A_w - S_P\right)\phi_P = \left(\frac{\Gamma_w}{\delta x_{WP}} A_w\right)\phi_W + \left(\frac{\Gamma_e}{\delta x_{PE}} A_e\right)\phi_E + S_U$$

or

$$\underline{a_P \phi_P = a_W \phi_W + a_E \phi_E + S_U}$$

where: $a_W = \frac{\Gamma_w}{\delta x_{WP}} A_w$, $a_E = \frac{\Gamma_e}{\delta x_{PE}} A_e$, $a_P = a_W + a_E - S_P$

$$\bar{S}\,\Delta V = S_U + S_P \phi_P$$

Finite Volume treatment of Sources

1. Constant known source $\dot{q}$: Set $S_p = 0$, $S_u = \dot{q} A \Delta x$
2. Source function of distance, e.g. $S = 5 - 20x$:

   Set $S_p = 0$, $S_u = (5 - 20x) A \Delta x$

   where x takes on a different value as we move in x.

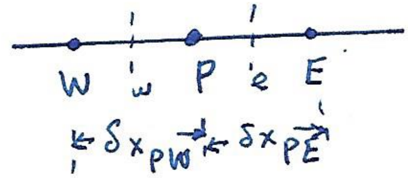
3. Source function of the dependent variable T:

   e.g. $S = a(T - b)$ where a, b constants:

   Set $S_p = a T A \Delta x$ and $S_u = -ab A \Delta x$

Finite Volume 1D equations

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) + S = 0$$



$$a_P \phi_P = a_E \phi_E + a_W \phi_W + S_U \quad (1)$$

where: $a_E = \dfrac{K_e A_e}{\delta x_{PE}}$, $a_W = \dfrac{K_w A_w}{\delta x_{WP}}$

$\bar{S}\Delta V = S_U + S_P \phi_P$ and I speify S_U, S_P

$$a_P = a_E + a_W - S_P$$

Boundary conditions: We cut the link (i.e either a_W or a_E, depending on where the boundary is) and we add the boundary flux through S_U, S_P

So for given value of T:

$$S_U = \frac{2K_b A_b}{\Delta x} T_b \quad , \quad S_P = -\frac{2K_b A_b}{\Delta x}$$

b stands for either W or E and T_b for the known b.c at b.