σχολήχημικώνμηχανικών
εθνικόμετσόβιοπολυτεχνείο

Συνήθεις Διαφορικές Εξισώσεις – Πρόβλημα Αρχικών τιμών

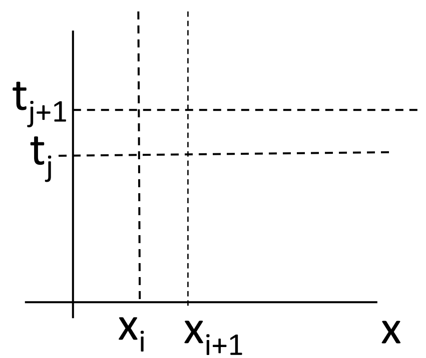
http://ecourses.chemeng.ntua.gr/courses/computational_methods_for_engineers/

Parabolic PDEs

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad u = u(t, x); \quad u(0, x) = a; \quad u(t, 0) = b; \quad u(t, 1) = c$$

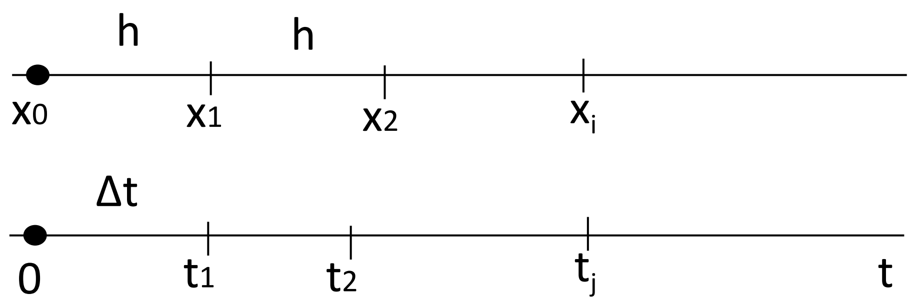
1. Διαμέριση ανεξάρτητων μεταβλητών

Spatial-temporal discretization:



$x_i = i\,h$

$t_j = j\,\Delta t$



2. Επιλογή σχήματος πεπερασμένων διαφορών

$$\left.\frac{\partial u}{\partial t}\right|_{t=t_j} \quad \frac{u(t_{j+1})-u(t_j)}{\Delta t}=\frac{u_{i,j+1}-u_{i,j}}{\Delta t}$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_i} \quad \frac{u(x_{i+1})-2u(x_i)+u(x_{i+1})}{h^2}=\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2}$$

3. Αντικατάσταση στη Δ.Ε.

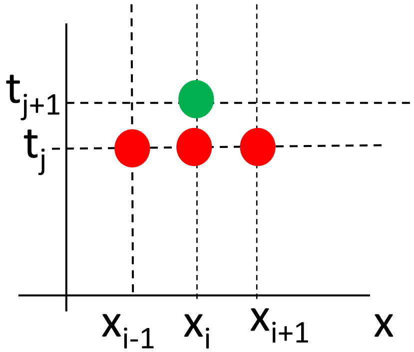
π.χ. $$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}=\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h^2}$$

i=1,2,…,N

j=1,2,…,M

4. Επίλυση συστήματος αλγεβρικών εξισώσεων

- explicit

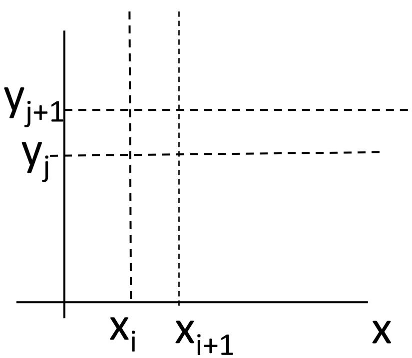


Ellipric PDEs

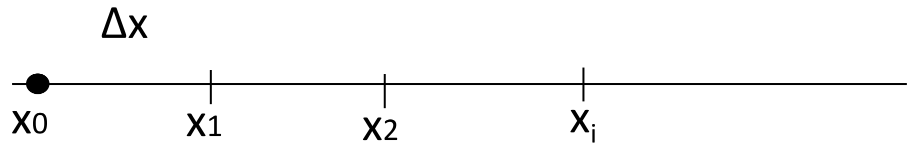
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y), \qquad u = u(x,y); \quad u(0,y) = a; u(1,y) = b \quad u(x,0) = c; u(x,1) = d$$

1. Διαμέριση ανεξάρτητων μεταβλητών

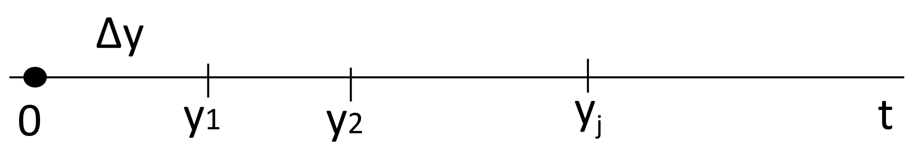
2D Spatial discretization:



$x_i = i\,\Delta x$



$y_j = j\,\Delta y$



2. Επιλογή σχήματος πεπερασμένων διαφορών

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_i} \quad \frac{u(x_{i+1}) - 2u(x_i) + u(x_{i+1})}{h^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{x=x_i} \quad \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2}$$

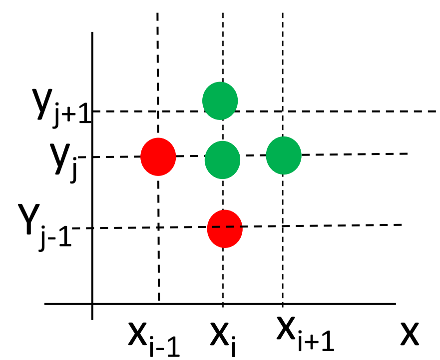
π.χ. $$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} = f(x_i, y_j)$$

$i=1,2,\ldots,N$

$j=1,2,\ldots,M$

4. Επίλυση συστήματος αλγεβρικών εξισώσεων

- implicit



5. Solution of an NxM linear system

Direct methods: Gauss elimination (A\b)

Indirect methods:

Jacobi Iterations:

a) Assume initial guess for u (i,j)
b) Get improved solution unew (I,j) from

$$\frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{\Delta x^2}+\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{\Delta y^2}=f(x_i,y_j)$$

c) Check convergence to stop

norm (unew-uold)<TOL

residual <TOL

Gauss-Seidel Iterations:

Same as Jacobi but unew is used immediately

SOR Iterations:

Successive over Relaxation

$u^{new}=u^{old}+\omega\,\Delta u$

Program (1)

```
C
C                         P.D.E.
C
C                        Jacobian
C
      DOUBLE PRECISION U(35,35),UN(35,35),RU(34,34),LARGE,Dx,Dy
      OPEN(UNIT=1,FILE='JACOBIAN.DAT')
      WRITE(*,*)'Input the mesh nxn, Re number ,TOLERANCE E'
      READ(*,*)N,Re,e
      Dx=2./(N+1)
      Dy=2./(N+1)
      K=N+2
      DO 20 i=1,K
      DO 15 j=1,K
      U(i,j)=0
   15 CONTINUE
   20 CONTINUE
      MAXITER=500
      ITER=0
C
   25 ITER=ITER+1
      ioutpt=0
C
      DO 35 i=2,N+1
      DO 30 j=2,N+1
      UN(i,j)=-1/(2/Dx**2+2/Dy**2)*(Re-(U(i-1,j)+U(i+1,j))/Dx**2-
     >(U(i,j-1)+U(i,j+1))/Dy**2)
   30 CONTINUE
   35 CONTINUE
C
      DO 38 i=2,N+1
      DO 37 j=2,N+1
      U(i,j)=UN(i,j)
   37 CONTINUE
   38 CONTINUE
C
      LARGE=-1
      DO 45 i=2,N+1
      DO 40 j=2,N+1
      RU(i,j)=-Re+(U(i-1,j)+U(i+1,j))/Dx**2+(U(i,j-1)+U(i,j+1))/Dy**2
     >-(2/Dx**2+2/Dy**2)*U(i,j)
      ABSRES=ABS(RU(i,j))
      IF (ABSRES.GT.e)  ioutpt=1
      LARGE=MAX(LARGE,ABSRES)
   40 CONTINUE
   45 CONTINUE
C
      WRITE(1,*) 'No of iteration',ITER,'    ||RU||=',LARGE
      IF (ITER.GT.MAXITER) GOTO 80
      IF (ioutpt.GT.0) THEN
      GOTO 25
      ELSE
      GOTO 100
      ENDIF
   80 Write(*,*) 'stopped becouse we exceed MAXITER'
  100 WRITE(1,*) 'JACOBIAN' ,'GRID=',N,'Re=',Re
      WRITE(1,*) 'TOLERANCE=',e,'ITERATIONS=',ITER
      WRITE(1,*)
      DO 110 i=2,N+1
      DO 105 J=2,N+1
      WRITE(1,*)U(i,j)
  105 CONTINUE
  110 CONTINUE
      CLOSE(UNIT=1)
      END
```